A New Stochastic Interpretation of Quantum Mechanics

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The reinterpretation of quantum mechanical formalism in terms of a classical model with a continuous material "Ψ -field" acting upon a point-like particle which is subjected to large friction and random forces is proposed. This model gives a mechanism for sudden "quantum jumps" and provides a simple explanation of "Schrödinger Cat" phenomena.

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Although the formalism of quantum mechanics and its predictions within the generally accepted limits of applicability are beyond any doubt the interpretation of the fundamental object– the wave function Ψ– is still a topic of debate at least for a small but dedicated group of physicist and philosophers [1]. The standard Copenhagen interpretation which states that $|\Psi(X,t)|^2$ is the probability density of finding at the time t the physical system in a point X of the configurations space produces an elegant mathematical formalism of Quantum Probability and a consistent theory with an enormous predictive power. Nevertheless the immaterial nature of the continuous wave function contrasted with a point-like structure of elementary particle reconciliated finally within the Bohr's "wave-particle dualism" disturbs from time to time a good frame of mind of not only undergraduate students but also professional physicists. Therefore the critical discussion of alternative interpretations is not completely pointless at least from the pedagogical point of view. Moreover there exists a class of phenomena which are difficult to reconcile with the orthodox first-principles quantum mechanics. One can mention in this context the following topics: the "problem of molecular structure" [2] in particular the existence of optical isomers, Schrödinger Cat paradox or more generally the absence of superpositions of macroscopically distinguishable states. The interpretation of quantum theory proposed in this note provides a simple explanation of these phenomena preserving at the same time the mathematical formalism of quantum mechanics as a basic ingredient. The main idea may look similarly to the ideas of Nelson [3] and Bohm interpretation of quantum mechanics [4] but in fact is quite different and essentially simpler. In contrast to Nelson stochastic mechanics which is mathematically equivalent to the standard quantum theory the presented formalism reproduces quantum mechanical predictions under certain additional conditions. Unlike in classical field theories (e.g. electrodynamics, gravitation theory) the fundamental object is not a space-time but a configuration space Ω of the physical system (possible the whole Universe) and an absolute (cosmological) time t as a parameter. Two different entities live on Ω: the wave function $\Psi(X,t), X \in \Omega$ and a point-like "Particle" described by the trajectory $X(t) \in \Omega$. Here Ψ is a really existing "classical" field which can be detected by its influence on the Particle. The time evolution is given by a pair of equations: the Schrödinger equation for Ψ

$$\frac{\partial}{\partial t}\Psi(X,t) = -\frac{i}{\hbar}\mathcal{H}\Psi(X,t) \tag{1}$$

where $\mathcal{H}$ is the Hamiltonian operator which contains all information about the structure of space-time and symmetries of the theory and the equation of motion describing the trajectory $X(t)$. The later depends on the structure of Ω and we restrict our discussion to the simplest case of $\Omega = \mathbf{R}^N$. In this case one can propose the following Langevin equation describing a Particle exercising a Brownian motion on Ω under the influence of an external time dependent potential $V(X,t) = -\ln|\Psi(X,t)|^2$

$$\frac{d}{dt}X(t) = \lambda\nabla_X \ln|\Psi(X(t),t)|^2 + \sqrt{2\lambda}\xi(t) \tag{2}$$

where $\xi(t) = \{\xi_1(t), ..., \xi_N(t)\}$ is a universal white noise satisfying

$$< \xi_k(t)\xi_l(s) >= \delta_{kl}\delta(t-s) \tag{3}$$

and with the diffusion constant λ which will take large enough value. The logarithmic form of the potential allows to separate degrees of freedom when Ψ possesses a product structure. A possible value of λ can be parameterized as follows

$$\lambda = \frac{l_\Omega^2}{\tau} \tag{4}$$

where l_Ω is a characteristic "length scale" on the configuration space (e.g. Bohr radius in the atomic physics) and τ is a very short universal time scale of diffusion (e.g. Planck unit of time). This is a fundamental difference in comparison with other "stochastic approaches" where the diffusion constant is proportional to $\hbar$ and hence "small". The rapid diffusion in our model describes in classical terms the so-called "quantum jumps" [5]. The possible mechanism producing the friction and random forces in Eq.(2) is not specified here but it can be related either to a kind of "vacuum fluctuations" or due to the interaction between the enormous number of degrees of freedom. The probability distribution of the Particle position $p(X,t)$ satisfies the Smoluchowski equation completely equivalent to Eq.(2)

$$\frac{\partial p}{\partial t} = \lambda \nabla_X [-\nabla_X (\ln |\Psi|^2) p + \nabla_X p] \ . \tag{5}$$

For a very large λ and if the potential $V(X,t) = -\ln |\Psi(X,t)|^2$ does not produce infinite or high practically impenetrable barriers between different regions of the configurations space Ω the probability distribution $p(X,t)$ relaxes rapidly to its temporal equilibrium value such that the following adiabatic approximation is valid

$$p(X,t) = Z^{-1} |\Psi(X,t)|^2 + o(\lambda^{-1}) \tag{6}$$

where Z is a normalizing constant ($|\Psi|^2$ need not to be normalized to 1). Hence up to the short time relaxation effects which can be unobservable under the conditions of above on $|\Psi|^2$ and λ all statistical predictions of quantum mechanical formalism concerning the functions of X are recovered. It seems that the information about others observables (e.g. momentum) are obtained analyzing the time evolution of wave packets in configuration space. Hence in this case all measurable properties of the physical system are determined by the wave equation (1).

The basic assumption concerning the absence of impenetrable barriers of the potential $-\ln |\Psi(X,t)|^2$ is reasonable for the microscopic systems for which the superpositions of different quickly oscillating eigenstates of the Hamiltonian smear out the nodes of $|\Psi|^2$ and allows the rapid relaxation of $p(X,t)$ to the value proportional to $|\Psi(X,t)|^2$. However, in many cases the above picture is false. In a standard interference experiment with a single particle the nodes of the wave function restrict the random trajectory of the particle to a vinicity of the local maximum of Ψ. Therefore the actual trajectory of the particle is rather localized and depends sensitively on the initial conditions. Nevertheless repeating the experiment with a large number of particles we recover again the interference pattern predicted by the standard quantum theory. Like for the standard theory any interaction with a measuring device introduces extra degrees of freedom to the configuration space and destroys the interference as expected.

A different situation is illustrated by the following simple example. Consider a physical system of a single degree of freedom described by the wave function which is a superposition of two Gaussians of the width a and separated by the distance $2b$. Then

$$\Psi(x) = \exp\{(x-b)^2/2a^2\} + \exp\{(x+b)^2/2a^2\} \ . \tag{7}$$

The relaxation time for the corresponding double-well potential $-\ln |\Psi|^2$ can be roughly estimated by the Kramers formula [6] for the escape time T from the well which in our case is given by ($b >> a$)

$$T \approx \frac{a^3}{\lambda b} \exp(b^2/a^2) \ . \tag{8}$$

The rapid increase of the relaxation time with the ratio b/a implies (for the large b/a) the localization of the probability distribution $p(x,t)$ at a given well. The similar arguments provide the plausible explanation of the observed localization effects in optical isomers or macroscopic systems.

One can imagine a few possible modifications of the presented approach. The configuration space Ω can be replaced by the phase-space Γ if one uses the phase-space formulation of the Eq.(1) and an obvious extension of the Eqs.(2,3,5) to Γ. The linearity of the Schrödinger equation is not a fundamental principle anymore and small nonlinear perturbations in particular describing the influence of the Particle on Ψ are conceivable.

In conclusion we summarize several appealing features of the discussed approach.

1) The proposal is rather conservative: the most important object to be calculated first is the quantum state $\Psi(X,t)$ interpreted here as a classical field on Ω.

2) The formalism is not equivalent to the quantum mechanics, in particular there should be possible to test experimentally the failures of the adiabatic approximation (see Eq.(6)).

3) The underlying diffusion process with a large diffusion constant provides a physical mechanism of sudden "quantum jumps" between different states of the system.

4) The absence of superpositions of macroscopically distinguishable states is easy to explain.

5) As the diffusion equation (4) is irreversible the time arrow is build into the formalism.

6) The inconceivable world of quanta is replaced again by the classical notions of fields and point particles living, however, on the multidimensional configuration space.

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